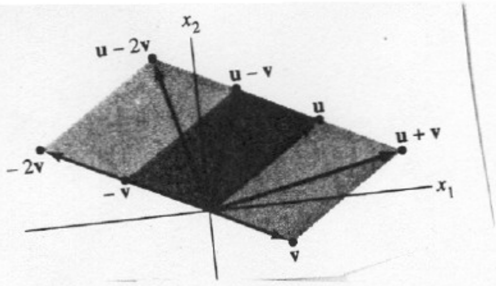
Section 1.3 & 1.4

4.



6. $x_1\begin{bmatrix}-2\\3\end{bmatrix}+x_2\begin{bmatrix}8\\5\end{bmatrix}+x_3\begin{bmatrix}1\\-6\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$, $\begin{bmatrix}-2x_1\\3x_1\end{bmatrix}+\begin{bmatrix}8x_2\\5x_2\end{bmatrix}+\begin{bmatrix}x_3\\-6x_3\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$, $\begin{bmatrix}-2x_1+8x_2+x_3\\3x_1+5x_2-6x_3\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$

$$\begin{aligned}-2x_2 + 8x_2 + x_3 &= 0\\ 3x_1 + 5x_2 - 6x_3 &= 0\end{aligned}$$

Usually the intermediate steps are not displayed.

10. $\begin{aligned}4x_1 + x_2 + 3x_3 &= 9\\ x_1 - 7x_2 - 2x_3 &= 2\\ 8x_1 + 6x_2 - 5x_3 &= 15\end{aligned}$, $\begin{bmatrix}4x_1+x_2+3x_3\\x_1-7x_2-2x_3\\8x_1+6x_2-5x_3\end{bmatrix}=\begin{bmatrix}9\\2\\15\end{bmatrix}$

$\begin{bmatrix}4x_1\\x_1\\8x_1\end{bmatrix}+\begin{bmatrix}x_2\\-7x_2\\6x_2\end{bmatrix}+\begin{bmatrix}3x_3\\-2x_3\\-5x_3\end{bmatrix}=\begin{bmatrix}9\\2\\15\end{bmatrix}$, $x_1\begin{bmatrix}4\\1\\8\end{bmatrix}+x_2\begin{bmatrix}1\\-7\\6\end{bmatrix}+x_3\begin{bmatrix}3\\-2\\-5\end{bmatrix}=\begin{bmatrix}9\\2\\15\end{bmatrix}$

Usually, the intermediate calculations are not displayed.

16. Some likely choices are $0\cdot\mathbf{v}_1+0\cdot\mathbf{v}_2=\mathbf{0}$, and

$1\cdot\mathbf{v}_1+0\cdot\mathbf{v}_2=\begin{bmatrix}3\\0\\2\end{bmatrix}$, $0\cdot\mathbf{v}_1+1\cdot\mathbf{v}_2=\begin{bmatrix}-2\\0\\3\end{bmatrix}$

$1\cdot\mathbf{v}_1+1\cdot\mathbf{v}_2=\begin{bmatrix}1\\0\\5\end{bmatrix}$, $1\cdot\mathbf{v}_1-1\cdot\mathbf{v}_2=\begin{bmatrix}5\\0\\-1\end{bmatrix}$

28. **a.** The amount of heat produced when the steam plant burns x_1 tons of anthracite and x_2 tons of bituminous coal is $27.6x_1 + 30.2x_2$ million Btu.

b. The total output produced by x_1 tons of anthracite and x_2 tons of bituminous coal is given by the vector $x_1\begin{bmatrix}27.6\\3100\\250\end{bmatrix}+x_2\begin{bmatrix}30.2\\6400\\360\end{bmatrix}$.

6. On the left side of the matrix equation, use the entries in the vector **x** as the weights in a linear combination of the columns of the matrix A:

$$-2\cdot\begin{bmatrix}7\\2\\9\\-3\end{bmatrix}-5\cdot\begin{bmatrix}-3\\1\\-6\\2\end{bmatrix}=\begin{bmatrix}1\\-9\\12\\-4\end{bmatrix}$$

8. The left side of the equation is a linear combination of four vectors. Write the matrix A whos
are those four vectors, and create a variable vector with four entries:

$$A = \left[\begin{bmatrix} 4 \\ -2 \end{bmatrix} \begin{bmatrix} -4 \\ 5 \end{bmatrix} \begin{bmatrix} -5 \\ 4 \end{bmatrix} \begin{bmatrix} 3 \\ 0 \end{bmatrix}\right] = \begin{bmatrix} 4 & -4 & -5 & 3 \\ -2 & 5 & 4 & 0 \end{bmatrix}, \text{ and } \mathbf{z} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix}. \text{ Then the equation}$$

is $\begin{bmatrix} 4 & -4 & -5 & 3 \\ -2 & 5 & 4 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} 4 \\ 13 \end{bmatrix}$.

12. To solve $A\mathbf{x} = \mathbf{b}$, row reduce the augmented matrix $[\mathbf{a}_1 \;\; \mathbf{a}_2 \;\; \mathbf{a}_3 \;\; \mathbf{b}]$ for the corresponding linear system:

$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ -3 & -1 & 2 & 1 \\ 0 & 5 & 3 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 5 & 5 & 1 \\ 0 & 5 & 3 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 5 & 5 & 1 \\ 0 & 0 & -2 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 5 & 5 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & 5 & 0 & -4 \\ 0 & 0 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & 1 & 0 & -4/5 \\ 0 & 0 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} \textcircled{1} & 0 & 0 & 3/5 \\ 0 & \textcircled{1} & 0 & -4/5 \\ 0 & 0 & \textcircled{1} & 1 \end{bmatrix}$$

The solution is $\begin{cases} x_1 = 3/5 \\ x_2 = -4/5 \\ x_3 = 1 \end{cases}$. As a vector, the solution is $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3/5 \\ -4/5 \\ 1 \end{bmatrix}$.

18. Row reduction shows that only three rows of B contain a pivot position:

$$B = \begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 1 & 2 & -3 & 7 \\ -2 & -8 & 2 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 0 & -1 & -1 & 5 \\ 0 & -2 & -2 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -7 \end{bmatrix} \sim \begin{bmatrix} \textcircled{1} & 3 & -2 & 2 \\ 0 & \textcircled{1} & 1 & -5 \\ 0 & 0 & 0 & \textcircled{-7} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Because not every row of B contains a pivot position, Theorem 4 in Section 1.4 shows that th
$B\mathbf{x} = \mathbf{y}$ does *not* have a solution for each $\mathbf{y}$ in $\mathbf{R}^4$.

34. If the equation $A\mathbf{x} = \mathbf{b}$ has a unique solution, then the associated system of equations does no
free variables. If every variable is a basic variable, then each column of A is a pivot column. S

reduced echelon form of A must be $\begin{bmatrix} \textcircled{1} & 0 & 0 \\ 0 & \textcircled{1} & 0 \\ 0 & 0 & \textcircled{1} \end{bmatrix}$. Now it is clear that A has a pivot position in

By Theorem 4, the columns of A span $\mathbf{R}^3$.